

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B:
1. All question are compulsory.
 2. Figures to the right indicate full marks.
 3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1 Attempt any THREE of the following:

- A) State whether following set of vectors are linearly dependent or linearly independent. If dependent find the relation between them. $X_1 = (1, 2, 3)$, $X_2 = (3, -2, 1)$, $X_3 = (1, -6, 5)$. 5
- B) Find the inverse of the matrix by adjoint method: 5
- $$A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$
- C) Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ into a diagonal matrix. 5
- D) Simplify $(1 + i)^{18}$. 5
- E) Prove that $\sin h^{-1} x = \log(x + \sqrt{x^2 + 1})$. 5
- F) Prove that $\text{Log}(1 + i) = \log \sqrt{2} + i(2n\pi + \pi/4)$. 5

Q.2 Attempt any THREE of the following:

- A) Solve $(x + 9y - 7)dx = (2x + 3y - 6)dy$. 5
- B) Solve $(y^2 + 2xy)dx + x^2 dy = 0$. 5
- C) Solve $(y \log y)dx + (x - \log y)dy = 0$. 5
- D) Solve $y^2 p^2 - 3xp + y = 0$. 5
- E) Solve $(D^2 - 3D - 4)y = 0$. 5
- F) Solve $(1 + y^2) + (x - \tan^{-1} y) \frac{dy}{dx} = 0$. 5

Q.3 Attempt any THREE of the following:

- A) Find the Laplace transform of $e^{2t} \cos^2 t$. 5
- B) Find the Laplace transform of $t^2 \sin at$. 5

- C) Use transform method to solve:
 $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$. 5
- D) Evaluate $\int_0^\infty te^{-3t} \sin t \, dt$. 5
- E) Find the inverse transforms of $\frac{s+7}{s^2+2s+5}$. 5
- F) Find the inverse Laplace transform $\frac{s^2}{(s+a)^3}$. 5

Q.4 Attempt any THREE of the following:

- A) Evaluate $\iint_S \sqrt{xy - y^2} \, dydx$, Where S is a triangle with vertices (0, 0), (10, 1) and (1, 1). 5
- B) Change the order of integrals and evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$. 5
- C) Evaluate $\int_0^\pi \int_0^{2\pi} \int_0^1 r^4 \sin \theta \, dr \, d\theta \, d\phi$. 5
- D) Evaluate the integral $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} \, dz \, dy \, dx$. 5
- E) Find the area enclosed by the pair of curves $y = 2 - x$ and $y^2 = 2(2 - x)$ using double integration. 5
- F) Find the volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. 5

Q.5 Attempt any THREE of the following:

- A) Evaluate $\int_0^\infty x^9 e^{-2x^2} \, dx$. 5
- B) Evaluate $\int_0^\infty \frac{x^4}{4^x} \, dx$. 5
- C) Evaluate $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}}$. 5
- D) Compute $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta$. 5
- E) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} \, dx$, where $a \geq 0$. 5
- F) Verify the rule of differentiation under integral sign for the integral $\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right) \, dx$. 5
